The Physics of Braking Systems
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Author's disclaimer: mechanical systems operating in the physical world are neither 100% efficient nor are they capable of instantaneous changes in state. Consequently, the equations and relationships presented herein are approximations of these braking system components as best as we understand their mechanizations and physical attributes. Where appropriate, several examples of limiting conditions and primary inefficiencies have been identified, but please do not assume these partial lists to be all-encompassing or definitive in their qualifications.

The Conservation of Energy

The braking system exists to convert the energy of a vehicle in motion into thermal energy, more commonly referred to as heat. From basic physics, the kinetic energy of a body in motion is defined as:

$$Kinetic\ Energy = \frac{1}{2} \times m_v \times v_v^{\,2}$$

- where m_v = the mass (commonly thought of as weight) of the vehicle in motion
- where v_v = the velocity (commonly known as speed) of the vehicle in motion

Ideally, this energy is completely absorbed by the braking system. While this is not entirely the case, for a stopping event at maximum deceleration most of the vehicle's kinetic energy is converted into thermal energy as defined by:

$$\frac{1}{2} \times m_v \times v_v^{\,2} \Rightarrow m_b \times C_p \times \Delta T_b$$

- where m_b = the mass of the braking system components which absorb energy
- where C_p = the specific heat of the braking system components which absorb energy (a constant based on material properties)
- where ΔT_b = the temperature rise experienced by the braking system components which absorb energy

Note that for most single-stop events, the rotors serve as the primary energy absorbing components.

It follows then that the temperature rise of the braking system is directly proportional to the mass of the vehicle in motion. More importantly perhaps, the temperature rise of the braking system is directly proportional to the *square* of the velocity of the vehicle in motion. In other words, doubling speed will theoretically quadruple brake temperatures:

In practical application, tire rolling resistance, aerodynamic drag, grade resistance, and other mechanical losses will also play an energy-absorbing role, but value is still placed in establishing this fundamental relationship as a limiting condition.

The Brake Pedal

The brake pedal exists to multiply the force exerted by the driver's foot. From elementary statics, the force increase will be equal to the driver's applied force multiplied by the lever ratio of the brake pedal assembly:

$$F_{bp} = F_d \times \{L_2 \div L_1\}$$

- where F_{bp} = the force output of the brake pedal assembly
- where F_d = the force applied to the pedal pad by the driver
- where L_1 = the distance from the brake pedal arm pivot to the output rod clevis attachment
- where L_2 = the distance from the brake pedal arm pivot to the brake pedal pad

Note that this relationship assumes 100% mechanical efficiency of all components in the brake pedal assembly. In practical application, the mechanical deflection of components and friction present in physical interfaces prevents this condition.

The Master Cylinder

It is the functional responsibility of the master cylinder to translate the force from the brake pedal assembly into hydraulic fluid pressure. Assuming incompressible liquids and infinitely rigid hydraulic vessels, the pressure generated by the master cylinder will be equal to:

$$P_{mc} = \frac{F_{bp}}{A_{mc}}$$

- where P_{mc} = the hydraulic pressure generated by the master cylinder
- where A_{mc} = the effective area of the master cylinder hydraulic piston

Note that this relationship assumes 100% hydraulic efficiency of all components in the master cylinder assembly. In practical application, fluid properties, seal friction, and compliance the physical components prevents this condition.

Brake Fluid, Brake Pipes, and Hoses

It is the functional responsibility of the brake fluid, brake pipes, and hoses to transmit the hydraulic fluid pressure from the master cylinder to the calipers located at the wheel ends. Out of necessity, part of this subsystem must be constructed from flexible (compliant) materials, as the wheel ends are free to articulate relative to the vehicle's unsprung mass (most commonly known as the body structure). However, again assuming incompressible liquids and infinitely rigid hydraulic vessels, the pressure transmitted to the calipers will be equal to:

$$P_{cal} = P_{mc}$$

- where P_{cal} = the hydraulic pressure transmitted to the caliper

Note that this relationship assumes 100% hydraulic efficiency of all brake fluid, brake pipes, and hoses. In practical application, fluid properties and the compliance inherent in flexible brake hoses prevent this condition.

The Caliper, Part I

It is the first functional responsibility of the caliper to translate the hydraulic fluid pressure from the pipes and hoses into a linear mechanical force. Once again assuming incompressible liquids and infinitely rigid hydraulic vessels, the one-sided linear mechanical force generated by the caliper will be equal to:

$$F_{cal} = P_{cal} \times A_{cal}$$

- where F_{cal} = the one-sided linear mechanical force generated by the caliper
- where A_{cal} = the effective area of the caliper hydraulic piston(s) found on one half of the caliper body

Note that this relationship assumes 100% hydraulic efficiency of all components in the caliper assembly. In practical application, fluid properties, seal friction, and compliance the physical components prevents this condition.

The Caliper, Part II

It is the second functional responsibility of the caliper to react the one-sided linear mechanical force in such a way that a clamping force is generated between the two halves of the caliper body. Regardless of caliper design (fixed body or floating body), the clamping force will be equal to, in theory, twice the linear mechanical force as follows:

$$F_{clamp} = F_{cal} \times 2$$

- where F_{clamp} = the clamp force generated by the caliper

Note that this relationship assumes 100% mechanical efficiency of all components in the caliper assembly. In practical application, mechanical deflection and, in the case of floating caliper bodies, the friction in the caliper slider assembly components prevents this condition.

The Brake Pads

It is the functional responsibility of the brake pads to generate a frictional force which opposes the rotation of the spinning rotor assembly. This frictional force is related to the caliper clamp force as follows:

$$F_{friction} = F_{clamp} \times \mu_{bp}$$

- where $F_{friction}$ = the frictional force generated by the brake pads opposing the rotation of the rotor
- where μ_{bp} = the coefficient of friction between the brake pad and the rotor

Note that this relationship assumes 100% mechanical efficiency of all components at the brake pad interface. In practical application, mechanical deflection (compressibility) of the brake pad materials and friction found between the brake pad and the caliper body components prevents this condition. In addition, it should be noted that the coefficient of friction between the brake pad and the rotor is not a single fixed value, but rather changes dynamically with time, temperature, pressure, wear, and such.

The Rotor

While the rotor serves as the primary heat sink in the braking system, it is the functional responsibility of the rotor to generate a retarding torque as a function of the brake pad frictional force. This torque is related to the brake pad frictional force as follows:

$$T_r = F_{friction} \times R_{eff}$$

- where T_r = the torque generated by the rotor
- where R_{eff} = the effective radius (effective moment arm) of the rotor (measured from the rotor center of rotation to the center of pressure of the caliper pistons)

Because the rotor is mechanically coupled to the hub and wheel assembly, and because the tire is assumed to be rigidly attached to the wheel, the torque will be constant throughout the entire rotating assembly as follows:

$$T_t = T_w = T_r$$

- where T_t = the torque found in the tire
- where T_w = the torque found in the wheel

Note that this relationship assumes 100% mechanical efficiency of all components at the wheel end. In practical application, mechanical deflection and relative motion between the rotating components prevents this condition.

The Tire

Assuming that there is adequate traction (friction) between the tire and the road to accommodate the driver's braking request, the tire will develop slip in order to react the torque found in the rotating assembly. The amount of slip generated will be a function of the tire's output characteristics (the mu-slip relationship), but the force reacted at the ground will be equal to:

$$F_{tire} = \frac{T_t}{R_t}$$

- where F_{tire} = the force reacted between the tire and the ground (assuming friction exists to support the force)
- where R_t = the effective rolling radius (moment arm) of the loaded tire

Up to this point our analysis has consisted of a single wheel brake assembly; however, because modern vehicles have one wheel brake assembly at each corner of the car, there are actually four tire forces being reacted during a typical stopping event. Because of this condition, the total braking force generated is defined as the sum of the four contact patch forces as follows:

$$F_{total} = \sum F_{tire\ LF,\ RF,\ LR,\ RR}$$

- where F_{total} = the total braking force reacted between the vehicle and the ground (assuming adequate traction exists)

Deceleration of a Vehicle in Motion

Based on the work of Sir Isaac Newton, if a force is exerted on a body it will experience a commensurate acceleration. Convention dictates that accelerations which oppose the direction of travel are called decelerations. In the case of a vehicle experiencing a braking force, the deceleration of the vehicle will be equal to:

$$a_v = \frac{F_{total}}{m_v}$$

- where a_v = the deceleration of the vehicle

Kinematics Relationships of Vehicles Experiencing Deceleration

Integrating the deceleration of a body in motion with respect to time allows for the determination of speed. Integrating yet again allows for the determination of position. Applying this relationship to a vehicle experiencing a linear deceleration, the theoretical stopping distance of a vehicle in motion can be calculated as follows:

$$SD_v = \frac{{v_v}^2}{2 \times a_v}$$

- where SD_v = the stopping distance of the vehicle

Note that this equation assumes a step-input deceleration from a fixed speed followed by a linear and constant rate of deceleration until the vehicle comes to rest. In practical application, deceleration cannot be achieved instantaneously, nor can deceleration be assumed to be constant for the duration of a stopping event.

Determining Parameters Related to Vehicle Static Weight Distribution

When either at rest or under conditions of zero acceleration, a vehicle will have a fixed distribution of mass (more commonly referred to as weight) which results in the four corners of the vehicle each suspending a fixed percentage of the total. In the side view, the sum of the left front and right front weights will equal the front axle weight and the sum of the left rear and right rear weights will equal the rear axle weight. If these values are known, then one can quickly calculate the static weight distribution as follows:

$$Percent\ \ front\ weight = \frac{V_f}{V_t} \times 100$$

and

$$Percent\ rear\ weight = \frac{V_r}{V_t} \times 100$$

- where V_f = the front axle vertical force (weight)
- where V_r = the rear axle vertical force (weight)
- where V_t = the total vehicle vertical force (weight)

If the static weight distribution is known, then calculating the longitudinal position of the vehicle's center of gravity (CG) is simply a function of geometry as follows:

$$CG_{f,x} = \frac{V_r}{V_t} \times WB$$

and

$$CG_{r,x} = \frac{V_f}{V_t} \times WB$$

- where $CG_{f,x}$ = distance from the front axle to the CG
- where $CG_{r,x}$ = distance from the rear axle to the CG
- where WB = the vehicle wheelbase (distance from the front axle to the rear axle)

From these relationships, it naturally follows that:

$$CG_{f,x} + CG_{r,x} = WB$$

Dynamic Impacts of Vehicles Experiencing Deceleration

Whenever a vehicle experiences a deceleration, the effective normal force (again, more commonly referred to as weight) reacted at the four corners of the vehicle will change.

While the total vehicle normal force remains constant, the front axle normal force during a deceleration event will increase while the rear axle normal force will decrease by the same amount. As the following equation demonstrates, the magnitude is a function of deceleration and vehicle geometry:

$$WT = \left(\frac{a_v}{g}\right) \times \left(\frac{h_{cg}}{WB}\right) \times V_t$$

- where WT = the absolute weight transferred from the rear axle to the front axle
- where g = the acceleration due to gravity (effectively expressing a_v in units of g's)
- where h_{CG} = the vertical distance from the CG to ground

In order to calculate the steady-state vehicle axle vertical forces during a given stopping event, the weight transferred must be added to the front axle static weight and subtracted from the rear axle static weight as follows:

$$V_{f,d} = V_f + WT$$

and

$$V_{r,d} = V_r - WT$$

- where $V_{f,d}$ = the front axle dynamic vertical force for a given deceleration
- where $V_{r,d}$ = the rear axle dynamic vertical force for a given deceleration

From these relationships, it naturally follows that for any given deceleration:

$$V_{f,d} + V_{r,d} = V_t$$

Effects of Weight Transfer on Tire Output

As a vehicle experiences dynamic weight transfer, the ability of each axle to provide braking force is altered. Under static conditions, the maximum braking force that an axle is capable of producing is defined by the following relationships:

$$F_{tires,f} = \mu_{peak,f} \times V_f$$

and

$$F_{tires,r} = \mu_{peak,r} \times V_r$$

- where $F_{tires,f}$ = the combined front tire braking forces
- where $F_{tires,r}$ = the combined rear tire braking forces
- where $\mu_{peak,f}$ = the maximum effective coefficient of friction between the front tires and the road
- where $\mu_{peak,r}$ = the maximum effective coefficient of friction between the rear tires and the road

However, as a result of weight transfer during a deceleration event the maximum braking force that an axle is capable of producing is modified as follows:

$$F_{tires,f} = \mu_{peak,f} \times V_{f,d} = \mu_{peak,f} \times (V_f + WT)$$

and

$$F_{tires,r} = \mu_{peak,r} \times V_{r,d} = \mu_{peak,r} \times (V_r - WT)$$

As shown by the relationships above, weight transfer increases the ability of the front axle to provide braking force while simultaneously decreasing the ability of the rear axle to provide braking force.

Note that in this analysis it is assumed that $\mu_{peak,f}$ and $\mu_{peak,r}$ are independent of deceleration, when in practice they are sensitive to the loading changes brought about by the weight transfer phenomenon. Consequently, as weight is transferred the total vehicle deceleration capability is diminished by a small amount.

Calculating Optimum Brake Balance

In order to achieve optimum brake balance, or to achieve 100% base brake efficiency, the ratio of the front and rear braking forces will be equal to the ratio of the front and rear vertical forces (axle weights). Under static conditions, this leads to:

$$\frac{F_{tires,f}}{V_f} = \frac{F_{tires,r}}{V_r}$$

However, as the brakes are applied the effects of weight transfer must be considered, as the ratio of front and rear vertical forces will change as follows:

$$\frac{F_{tires,f}}{V_{f,d}} = \frac{F_{tires,r}}{V_{r,d}}$$

From this relationship it becomes apparent that while the ratio of the front and rear braking forces is a fixed parameter based on the mechanical sizing of the brake system components, the ratio of the front and rear vertical forces is a variable based on deceleration and vehicle geometry. This dictates that relationship can only optimized for only one vehicle deceleration level and loading condition (typically at maximum deceleration with the highest percentage of static front weight).

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